Famous Statements in Number Theory

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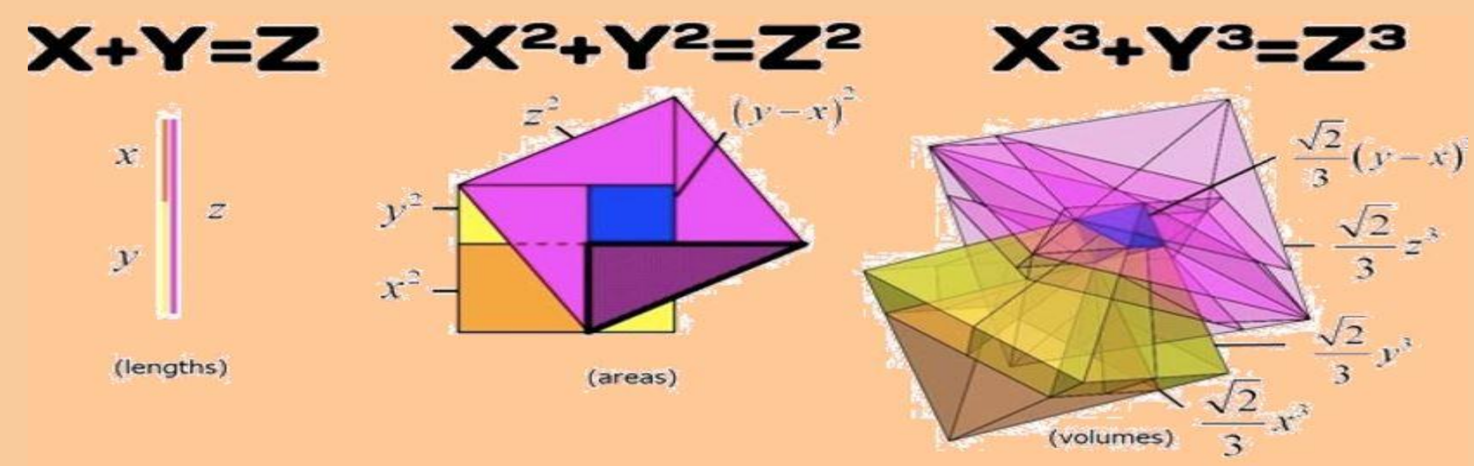
Fermat's Last Theorem

Fermat's Last Theorem is the statement that there are no natural numbers x, y, z, and n>2, such that

$$x^n + y^n = z^n$$

Fermat first described this as a conjecture in 1637, and it baffled mathematicians for centuries. At one point, some claimed it was false and unprovable; however, in 1995, a proof was found by Andrew Wiles after seven years of hard work.



As seen in the picture, n=1 and n=2 work, but n=3 doesn't.

Twin Primes Conjecture

The twin primes conjecture states that there exist infinitely many primes p, such that p+2 is also prime. It is unknown if there are infinitely many twin primes or if there is a largest pair.

In recent years, there has been progress towards solving this conjecture. For instance, Terry Tao from UCLA, considered the greatest mathematician alive, proved that there must exist either infinitely many twin primes (two steps apart), cousin primes (four steps apart), or sexy primes (six steps apart).

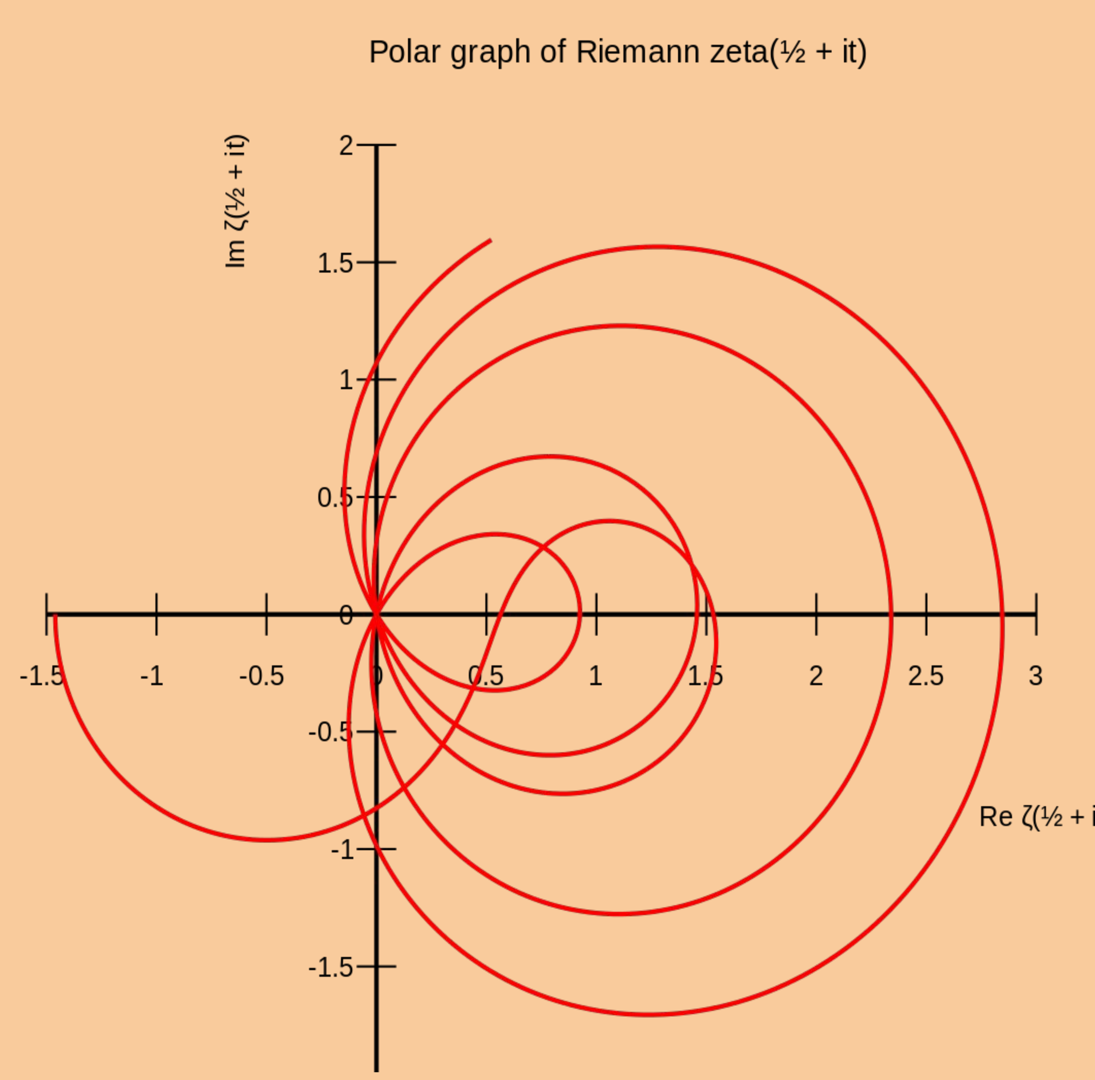
On the Colbert Show, Tao (pictured above) discusses winning the 2014 Breakthrough Prize in Mathematics, for which he earned a $3 million prize.

Riemann's Hypothesis

First proposed in 1859, the Riemann hypothesis states that the only zeroes (roots) of the Riemann zeta function are negative even integers (called trivial zeros) and complex numbers that have a real part of ½ (called non-trivial zeros).

The Riemann zeta function is defined to the right. Analytic continuation allows us to extend the original, limited definition to include complex and negative numbers.

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

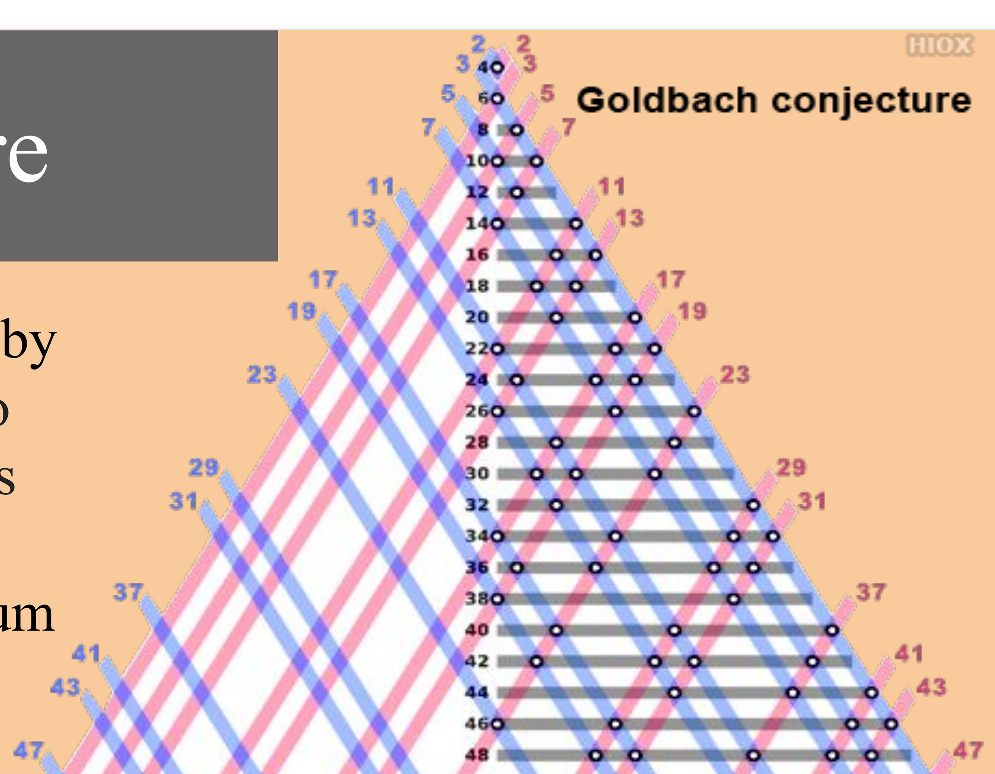


The Riemann hypothesis affects many other fields of math. Numerous results in number theory depend on this hypothesis, including the distribution of prime numbers, which might seem totally unconnected to this at first glance. It even played an important role in solving the odd version of the Goldbach conjecture. The Riemann hypothesis is considered a Millenium Problem, and there is a $1 million prize for solving it.

This is an incomplete graph of the non-trivial outputs of the function in the complex plane as the complex part of the input increases. Notice it passes through the origin in a cyclic way; each of these passes represent a zero of the function.

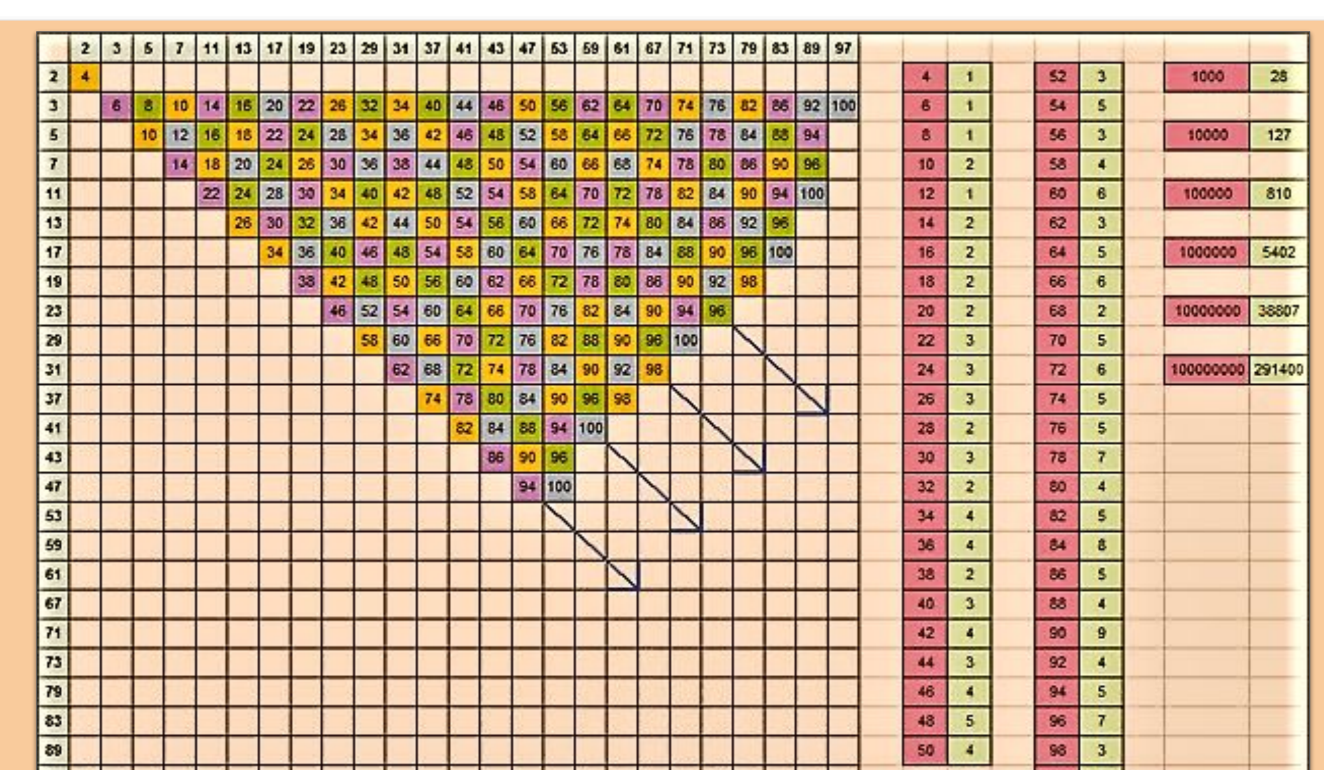
Goldbach's Conjecture

Goldbach's conjecture was proposed in 1742 by Russian mathematician Christian Goldbach to mathematician Leonhard Euler, and it remains unproven. Goldbach claimed that every even number greater than 2 can be written as the sum of two prime numbers. The greatest number proven to be true is 4* 10^18.



The diagram to the left shows that even numbers greater than 2 and less than 50 are a sum of two prime numbers ranging from 2 to 47, which ultimately proves Goldbach's conjecture to be true for all even numbers greater than 2 and less than or equal to 50.

Examples:
250 = 239 + 11 (239 & 11 are prime)
10000= 9941 + 59 (9941 & 59 are prime)



The diagram to the left, which was created using Excel, is a representation of Goldbach's conjecture being proven for even numbers greater than 2 and less than or equal to 100 using prime numbers from 2 to 97.

[Hungarian mathematician Paul Erdos, when asked why numbers are beautiful]
"It's like asking why is Ludwig van Beethoven's Ninth Symphony beautiful. If you don't see why, someone can't tell you. I know numbers are beautiful. If they aren't beautiful, nothing is."

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